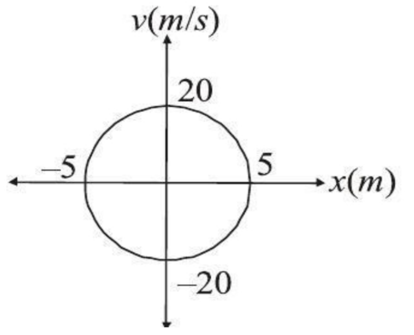
Oscillations

1. A rod of mass ' M ' and length ' $2L$ ' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of ' m ' are attached at distance ' $L/2$ ' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is

2. A simple pendulum of length 1 m is oscillating with an angular frequency $10 \mathrm{rad/s}$. The support of the pendulum starts oscillating up and down with a small angular frequency of $1 \mathrm{rad/s}$ and an amplitude of 10^{-2} m. The relative change in the angular frequency (in $\mathrm{rad/s}$) of the pendulum is given by :

3. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time (in second) it will take to drop to $\frac{1}{1000}$ of the original amplitude is

4. Figure shows $v - x$ graph of a particle executing simple harmonic oscillations. What is the velocity (in $\mathrm{m/s}$) of the particle at $x = 3$ m ?



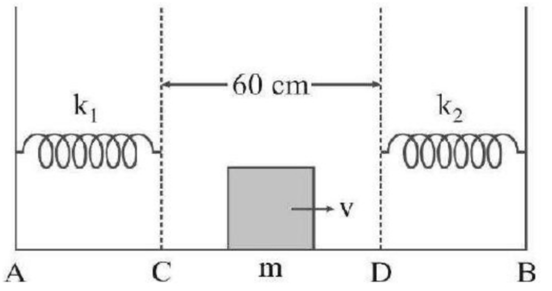
5. Two simple pendulums of length 1 m and 1.21 m are started oscillating from some position. Find the minimum time (in second) after which they again start from same position.

6. A spring balance has a scale that reads 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight (in Newton) of the body?

7. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. The ratio K_2/K_1 is

8. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t} \cdot \cos(10\pi t + \phi)$. Here t is in seconds. The time (in second) taken for its amplitude of vibration to drop to half of its initial value is

9. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid support and the other end is free as shown in figure. The distance CD between the free ends of springs is 60 cm. If the block moves along AB with a velocity $120 \mathrm{~cm/s}$ in between the springs, calculate the period (in second) of oscillation of block. ($k_1 = 1.8 \mathrm{~N/m}, k_2 = 3.2 \mathrm{~N/m}, m = 200$ g)



10. Two particles execute simple harmonic motion of same amplitude and frequency along the same straight line. They pass one another, when going in opposite directions, each time their displacement is half of their amplitude. What is the phase difference (in degree) between them ?

11. A cubical body (side 0.1 m and mass 0.002 kg) float in water. It is pressed and then released, so that it oscillates vertically. Find the time period (in second).

12. Two particles execute SHM with same frequency and amplitude along the same straight line. They cross each other, at a point midway between the mean and extreme position. Find the phase difference (in degree) between them.

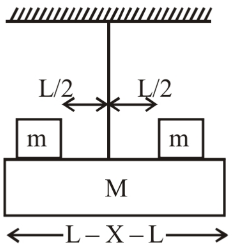
13. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72, find the maximum amplitude (in metre) of the table in which the block does not slip on the surface. $(g = 10 \text{ m/s}^2)$:

14. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5 \text{ T}/3$. Then the ratio of m/M is

15. A pendulum has time period T in air. When it is made to oscillate in water, it acquired a time period $\text{T}' = \sqrt{2}$ T. The specific gravity of the pendulum bob is equal to :

SOLUTIONS

1. **(0.37)**



$$f_1 = \frac{1}{2\pi}\sqrt{\frac{C}{1}} \qquad ...(i)$$

$$= \frac{1}{2}\sqrt{\frac{3C}{ML^2}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{C}{L^2\left(\frac{M}{3}+\frac{M}{2}\right)}} \qquad ...(ii)$$

As frequency reduces by 80%

$\therefore\ f_2 = 0.8\, f_1 \Rightarrow \frac{f_2}{f_1} = 0.8$ ...(iii)

Solving equations (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

2. **(10^{-3})** Angular frequency of pendulum $\omega = \sqrt{\frac{g}{\ell}}$

$\therefore$ relative change in angular frequency

$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g}{g}$ [as length remains constant]

$\Delta g = 2A\omega_s^2$ [ω_s = angular frequency of support and, A = amplitude]

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\times\frac{2A\omega_s^2}{g}$$

$$\Delta\omega = \frac{1}{2}\times\frac{2\times 1^2\times 10^{-2}}{10} = 10^{-3}\text{ rad/sec.}$$

3. **(20)** Time of half the amplitude is = 2*s*

Using, $A = A_0 e^{-kt}$

$$\frac{A_0}{2} = A_e\, e^{-k\times 2} \qquad ...(i)$$

and $\frac{A_0}{1000} = A_e\, e^{-kt}$ ...(ii)

Dividing (i) by (ii) and solving, we get

$t \simeq 20$ s

4. **(16)** Given, $v_{max} = \omega A = 20$ m/s and $A = 5$ m/s

$\therefore\ \omega = 4$ rad/s

Using $v = \omega\sqrt{A^2 - x^2}$

$= 4\sqrt{5^2 - 3^2} = 16$ m/s.

5. **(22)** Time period of first pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\,s$$

And time period of second pendulum,

$$T' = 2\pi\sqrt{\frac{1.21}{\ell}}$$

$$= 1.1 \times 2\pi\sqrt{\frac{1}{g}}$$

$$= 1.1\,T$$

If t is the required time and they complete n and m oscillations respectively, then

$$nT = mT'$$

or $$nT = m \times 1.1\,T$$

or $$\frac{n}{m} = 1.1$$

$$= \frac{11}{10}$$

So in the duration first pendulum complete 11 oscillations, second pendulum will complete 10 oscillations. Thus

$$t = n \times T = 2 \times 11 = 22\,s.$$

6. **(219.13)** Force constant of the spring,

$$k = \frac{Mg}{x} = \frac{50 \times 9.8}{0.20} = 2450\ N/m$$

The time period of oscillations is given by $T = 2\pi\sqrt{\frac{M}{k}}$

$\therefore$ $$M = \frac{T^2 k}{4\pi^2} = \frac{(0.60)^2 \times 2450}{4\pi^2}$$

$$= 22.36\ \text{kg}$$

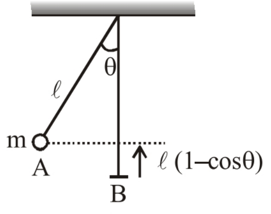
Weight of the body $= Mg = 22.36 \times 9.8$

$= 219.13\,N.$

7. **(2)** Maximum kinetic energy at lowest point B = Change in potential energy

i.e. $K = mgl\,(1 - \cos\theta)$

where θ = angular amp.



$K_1 = mg\ell\,(1 - \cos\theta)$ ...(i)

$K_2 = mg\,(2\ell)\,(1 - \cos\theta)$ ...(ii)

Solving (i) and (ii) we get

$K_2 = 2K_1$.

8. **(7)** Amplitude of vibration at time t = 0 is given by $A = A_0 e^{-0.1 \times 0} = 1 \times A_0 = A_0$

also at t = t, if $A = \frac{A_0}{2}$

$$\Rightarrow \frac{1}{2} = e^{-0.1t}$$

$t = 10 \ln 2 \simeq 7s$

9. **(2.83)** In the device the block remains in contact with first spring for half the time period. i.e., $\frac{T_1}{2}$. Similarly with the second spring it is $\frac{T_2}{2}$. If t is the time of motion from C to D, then total time of motion

$$T = 2t + \frac{T_1}{2} + \frac{T_2}{2}$$

$$= 2 \times \frac{CD}{v} + \frac{1}{2}\left[2\pi\sqrt{\frac{m}{k_1}} + 2\pi\sqrt{\frac{m}{k_2}}\right]$$

$$= 2.83 \text{ s}$$

10. (120) Suppose θ is the phase difference between them,

then $x_1 = A \sin \omega t$ ...(i)

and $x_2 = A \sin (\omega t + \phi)$ ...(ii)

Putting $x_1 = x_2$

$= A/2$ in the above equations, we get

$$\frac{A}{2} = A \sin \omega t$$

or $\sin \omega t = \frac{1}{2}$

and $\frac{A}{2} = A[\sin \omega t \cos \phi + \cos \omega t \sin \phi]$

or $\frac{A}{2} = A\left[\frac{1}{2}\cos\phi + \sqrt{1-\left(\frac{1}{2}\right)^2}\sin\phi\right]$

or $2\cos^2\phi - 2\cos\phi - 2 = 0$

$\therefore$ $\cos\phi = 1$ or $-1/2$

$\cos\phi = 1$ is not acceptable.

$\therefore$ $\cos\phi = -\frac{1}{2}$

or $\phi = 120°$

11. (0.028) The time period is given by

$$T = 2\pi\sqrt{\frac{M}{A\rho g}}$$

$$= 2\pi\sqrt{\frac{0.002}{0.1^2 \times 1000 \times 9.8}}$$

$$= 0.028\ s$$

12. (120) For first particle

$x = A \sin \phi_1$

or $\frac{A}{2} = A \sin \phi_1 \Rightarrow \sin\phi_1 = \frac{1}{2}$ or $\phi_1 = \frac{\pi}{6}$

For second particle,

$$\frac{A}{2} = A \sin \phi_2 = \frac{\pi}{6} \text{ or } \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

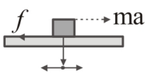
As $\phi_1 \neq \phi_2$ $\therefore$ $\phi_2 = \frac{5\pi}{6}$.

Now $\phi_2 - \phi_1 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$

13. (0.02) For no slip on table

$ma \leq f_{lim}$

or $m\omega^2 A \leq \mu mg$



$\therefore$ $A_{max} = \frac{\mu g}{\omega^2}$

$= 0.02\text{m}$

14. (1.78) $T = 2\pi\sqrt{\frac{M}{k}}$ and $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

After simplifying above equations, we get

$$\frac{m}{M} = \frac{16}{9}$$

15. **(2)** The effective acceleration of a bob in water $= g'$ $= g\left(1-\frac{d}{D}\right)$, where d and D are the density of water and the bob respectively. $\frac{D}{d}$ = specific gravity of the bob.

Since the period of oscillation of the bob in air and water are given as

$$T = 2\pi\sqrt{\frac{\ell}{g}} \text{ and } T' = 2\pi\sqrt{\frac{\ell}{g'}}$$

$$\therefore \quad T/T' = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g(1-d/D)}{g}} = \sqrt{1-\frac{d}{D}} = \sqrt{1-\frac{1}{s}}$$

Putting $T/T' = 1/\sqrt{2}$, we obtain

$$\frac{1}{2} = 1-\frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{s} = \frac{1}{2} \Rightarrow s = 2.$$